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CALIBRATIONS OF THE LINE STANDARDS OF LENGTH OF THE NATIONAL BUREAU OF STANDARDS

Lewis V. Judson and Benjamin L. Page

ABSTRACT

The results of the intercomparison of total lengths of meter and of decimeter bars, and the results of calibrations of subintervals of several of them are given in considerable detail. The precision of the results is discussed. That the basis of length measurements in this country (United States Prototype Meter 27) has remained unaltered to about a part in 10,000,000 for over 40 years is pointed out, and the wisdom of the conclusions stated by Louis A. Fischer in the first scientific paper published by the Bureau is shown.

CONTENTS

I. THE LINE STANDARDS OF LENGTH

Three groups of line standards of length are discussed in this paper, namely, platinum-iridium prototype meters, nickel-steel meters, and decimeters. By a line standard is meant a standard in which the length is defined as the distance between lines or graduations on the surface. Thus the platinum-iridium prototype meters are standards in which the meter, as a unit of length, is the nominal distance between specified lines on these bars. Such terms as meter and decimeter are commonly used to designate either a standard or a unit. Which of these two meanings is intended must be clear from the context. A complete calibration of a line standard involves a determination of its length at one temperature, a determination of its thermal expansivity, and a series of measurements to determine the corrections to subdivisions.

Instead of giving the length of a standard, it is more convenient to state the correction. The actual length is equal to the nominal length plus the correction. Hence a plus correction indicates that a standard is longer than the nominal length.

Unfortunately, the length of a standard at a specified temperature cannot be assumed to remain the same indefinitely, because the "aging" of metals commonly involves structural changes which alter the length. If these changes are not uniform throughout the length, a calibration of the subintervals of a standard before the changes took place will not give the true lengths afterward. A fundamental part of the work of the length-measuring laboratory of a national standardizing bureau is the repeated calibration of its standards so as to maintain the requisite accuracy at all times. An account of some of this work forms the basis of this paper.

1. PLATINUM-IRIDIUM METER BARS

The National Bureau of Standards (NBS) has four platinum-iridium meter bars obtained from the Bureau International des Poids et Mesures (BIPM) and certified by that Bureau. One of these, United States Prototype Meter 27, is considered to be the primary standard of length in the United States. Meter 21 is the meter bar most frequently used in determining the total length of the laboratory standards of length of the NBS. Meters 12 and 4, both of the "Alloy of 1874", have been used in less important comparisons and as checks. Meter 4 is subdivided and has been used considerably in standardizing lengths of less than a meter.

All of these bars are of the same X-shape as the International Prototype Meter.[1] With the exception of meter 4, which is subdivided to millimeters, each of these bars has near each end a group of three transverse lines intersected by two longitudinal lines. Measurements are made on the central one of each group of transverse lines, using that part of the line between the two longitudinal lines.

The lengths of these meters were determined at the BIPM before the bars were sent to this country. Meter 4 was received in 1907; the other three in 1890. Meter 27 was recompared at the BIPM in 1903 and again in 1922–23. Meter 21 was recompared there in 1932.

For a number of years the BIPM has been engaged in making two extensive series of measurements, one to determine the values of the coefficients of thermal expansion of the platinum-iridium meters more accurately than was possible when the original certificates were issued; the second, to obtain new intercomparisons of the total lengths of a large number of these meter bars. As a result of this work, new corrections at 0 °C and new coefficients of expansion were authorized by the Eighth General (International) Conference on Weights and Measures which met in Paris in 1933. In the case of meters 12 and 4 of the Alloy of 1874 which had not been returned to the BIPM the new values merely correct the computations made at the time of the original measurements, using the new coefficients and, in the case of meter 4, employing the probable value of laboratory standards used at the BIPM in 1907.

The values in the original certificates and also the new values are given in table 1.

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[1] For a more detailed description see p. 2 of BS Circular 332; p. 742 of Appendix 18, Report of USC&GS for 1890; or publications of the BIPM.

TABLE 1.—*Data concerning the meter and decimeter bars*

Length of bar at temperature T=[nominal length+correction at 0°C] × [1+(coefficient of expansion) ×T°]

NOTE.—Meters 27 and 21 are of the principal national series of prototypes; meters 12 and 4 are of the alloy of 1874; meters 39,153 R and 354 have serial numbers of the Société Genevoise. Meter 153 R has an inlaid strip of platinum for the graduations.

1	2	3	4	5	6	7	8	9	10
Designation of bars	Composition	Authority	Date	Correction at 0°C	Coefficient of expansion [b]	Authority	Date	Correction at 0°C	Coefficient of expansion
Meter:				μ				μ	
27	Platinum-iridium	BIPM	1889	−1.6	$(8.657+0.00100\ T)10^{-6}$	BIPM	1933	−1.47	$(8.6210+0.00180\ T)10^{-6}$.
21	do	BIPM	1889	+2.5	$(8.665+0.00100\ T)10^{-6}$	BIPM	1933	+3.42	$(8.6210+0.00180\ T)10^{-6}$.
12	do	BIPM	1889	+3.3	$(8.634+0.00100\ T)10^{-6}$	BIPM	1933	[a] +3.34	$(8.6014+0.00180\ T)10^{-6}$.
4	do	BIPM	1907	−5.9	$(8.655+0.00100\ T)10^{-6}$	BIPM	1933	[a] −5.13	$(8.6014+0.00180\ T)10^{-6}$.
39	Invar	NBS	1907		1.48×10^{-6}	NBS	1913		1.54×10^{-6}.
153 R	42-percent nickel steel					NBS	1914		7.02×10^{-6}.
354	Invar					NBS	1924		1.68×10^{-6}.
Decimeter:									
10	Invar	BIPM		+2.0	$(0.965-0.00017\ T)10^{-6}$	BIPM	1929		$(1.071-0.00181\ T)10^{-6}$.
20	do	BIPM		−4.5	$(0.965-0.00017\ T)10^{-6}$	BIPM	1929		$(1.071-0.00181\ T)10^{-6}$.
24	44-percent nickel steel	BIPM	1900	−1.8	$(8.298-0.00328\ T)10^{-6}$	BIPM	1929		$(8.273-0.00251\ T)10^{-6}$.
43	Invar	BIPM	1900	−4.0	$(0.965-0.00017\ T)10^{-6}$	BIPM	1929		$(1.071-0.00181\ T)10^{-6}$.
56	do	BIPM		+.4	$(0.965-0.00017\ T)10^{-6}$	BIPM	1929		$(1.071-0.00181\ T)10^{-6}$.
57	44-percent nickel steel	BIPM	1923	−6.0	$(8.298-0.00328\ T)10^{-6}$	BIPM	1929		$(8.273-0.00251\ T)10^{-6}$.
61	Nickel	BIPM	1923	−3.2	$(12.540+0.00635\ T)10^{-6}$	BIPM	1929		$(12.515+0.00712\ T)10^{-6}$.

[a] Value as recomputed from original observations prior to issuing original certificates.

[b] Certificates from the BIPM have given equations and temperatures on the basis of the normal hydrogen scale, while comparisons of length standards at NBS have been on the basis of the international temperature scale. These two scales are the same within the limits of reproducibility of the normal hydrogen scale. No distinction between the two temperature scales is made in this paper.

2. LABORATORY STANDARD METER BARS

At the NBS a group of meter bars of nickel steel, having the H-shape cross section, and made by the Société Genevoise d'Instruments de Physique, are used in many of the routine length comparisons. These bars are standardized by comparison with the platinum-iridium bars.

Two of these, meter 39 of invar and meter 153 R of 42 percent nickel steel with a dovetailed strip of platinum for the graduations, and also a meter, 354, of invar submitted to the NBS for test, are included in this intercomparison.

Meter 39 has had extensive use, and its decimeter intervals were used in the standardization of the decimeter bars described in this paper.

The coefficients of expansion determined for these bars are given in table 1.

3. DECIMETER BARS

The NBS has four decimeter bars obtained from the BIPM: a 44 percent nickel-steel bar, 24, and an invar bar, 43, both purchased in 1901, and a 44 percent nickel-steel bar, 57, and a pure nickel bar, 61, both purchased in 1923.

Three additional decimeter bars belonging to universities in this country were also made available for inclusion in the measurements. These bars are decimeter 10, belonging to Northwestern University; decimeter 20, belonging to Clark University; and decimeter 56, belonging to the University of Chicago.

Each of these bars has a rectangular cross section about 21 mm wide and 7 mm high, an extreme length of approximately 150 mm, a scale 100 mm long divided in millimeters, and a separate millimeter interval divided in tenths. The graduations are intersected by two longitudinal lines 0.2 mm apart, which define the part of the line to be used in making a micrometer setting.

The coefficients of thermal expansion given in the BIPM certificates for these decimeter bars were obtained in a test of 1-meter bars from the same melt, measurements being made at six different temperatures, ranging from 0 to 38° C. Since the certificates were issued, additional measurements have been made on the several 1-meter bars referred to above and new values for the coefficients reported. The lengths and both sets of coefficients are given in table 1.

According to the certificates, the errors of the divisions of these scales were determined by the aid of a longitudinal comparator especially constructed for that purpose. This comparator, constructed by Bariquand and Marre, is described in more detail in various publications of the International Bureau. In some of the calibrations, microscopes were used having a magnification of about 200, but in other cases the magnification was much less. The procedure for the measurements was this: After the scale had been placed in position in the comparator in alinement with another standard of the same metal, each of the centimeter intervals of the first standard was compared with each of the centimeter intervals of the second standard. Then each of the millimeter intervals of the first centimeter on one standard was compared with each of the millimeter intervals of the first centimeter of the second standard. The same procedure was followed for the intervals of the subdivided millimeter. This subdivided millimeter in turn was compared with 10 calibrated millimeter intervals of the auxiliary scale.

Observations at the NBS over a period of years on bars 24, 43, 57, and 61 indicated that the values certified by the BIPM were not consistent and that a complete intercomparison was advisable.

II. CALIBRATION OF THE METER BARS

The fundamental measurements in this work on the length standards were those made in 1928 in the intercomparison of the four platinum-iridium meters and invar meter 39. The results of an auxiliary comparison, made in 1930 primarily for another purpose, are included in this paper because they serve as a check on the earlier measurements. A subsidiary but important part of the work with the meter bars was the calibration of the invar meter to decimeters.

1. DETERMINATION OF TOTAL LENGTH

The intercomparison of the total length of the meter bars was undertaken at the NBS in 1928, using the new longitudinal comparator.[2] The bars were placed in the eight possible positions relative to each other, and 10 comparisons were made in each position at an average temperature of approximately 23° C. From these observations, values at 0° C were computed. As the recent BIPM determinations of coefficients of expansion for the platinum-iridium meter bars may be considered as fully meeting the needs of metrology, they may be used with confidence in reducing the above observations to 0° C. The results are given in table 2.

TABLE 2.—*Differences in length of meter bars at 0° C*

[1928 series of comparisons]

Observations made at temperature T and reduced to 0° C using values of the coefficients given in column 10 of table 1.
A−B=ΔL.

Bars compared	Observed difference, ΔL	Probable error	Observed temperature, T
A B	μ	μ	°C
21 minus 27	+5.088	±0.048	22.27
12 minus 27	+5.049	±.042	22.58
4 minus 27	−3.958	±.042	22.90
39 minus 27	−40.544	±.040	22.22
12 minus 21	−.113	±.043	24.01
4 minus 21	−8.862	±.040	23.17
39 minus 21	−45.638	±.043	22.57
4 minus 12	−8.754	±.032	23.47
39 minus 12	−45.421	±.036	23.58
39 minus 4	−36.766	±.049	22.40

The reduction of these observations by the method of least squares may be conveniently carried out by the use of the form shown in table 3.

[2] George K. Burgess, *Precision machines and instruments for the measurement of length,* Proc. World Eng. Cong., Tokyo, **5**, 6 (1929); paper 335.

TABLE 3.—*Calculation of corrections to meter bars at 0° C*

[1928 series of comparisons]

Observational values of (A−B) are taken from table 2.

Calculated ("most probable") values of (A−B), given in parentheses, are obtained by taking differences of Σ/5, e. g., meter 4−meter 12=+3.038−(+11.822)=−8.784 μ and meter 21−meter 27=+11.940−(+6.873)=+5.067 μ.

Correction to meter 27=−1.47 μ at 0° C. (See table 1.)

Correction to bar=correction to meter 27+(bar−27).

Length of bar=1 m+correction.

B \ A	27	21	12	4	39
	μ	μ	μ	μ	μ
27	0	+5.088 (+5.067)	+5.049 (+4.949)	−3.958 (−3.835)	−40.544 (−40.547)
21	−5.088	0	−0.113 (−0.118)	−8.862 (−8.902)	−45.638 (−45.614)
12	−5.049	+0.113	0	−8.754 (−8.784)	−45.421 (−45.496)
4	+3.958	+8.862	+8.754	0	−36.766 (−36.712)
39	+40.544	+45.638	+45.421	+36.766	0
Σ	+34.365	+59.701	+59.111	+15.192	−168.369
Σ/5	+6.873	+11.940	+11.822	+3.038	−33.674
Bar−27	0	+5.067	+4.949	−3.835	−40.547
Corr. to bar	−1.47	+3.60	+3.48	−5.31	−42.02

In 1930 an intercomparison of meter bars was made by three observers, none of whom had taken part in the 1928 measurements. This new series included the two platinum-iridium meters, 12 and 21, the invar meter 39, the 42 percent nickel-steel meter 153 R, and the invar meter 354. The results of these measurements are shown in table 4.

TABLE 4.—*Calculation of corrections to meter bars at 0° C.*

[Auxiliary series of comparisons, 1930]

Observational values of (A−B) are taken from the original data sheets.
Calculated ("most probable") values of (A−B), given in parentheses, are obtained by taking differences of Σ/5, e.g. meter 39−meter 12=28.45−(+16.88)=−45.33 μ
Correction to meter 21=+3.60 μ at 0° C. (See table 3.)
Correction to bar=correction to meter 21+(bar−21).
Length of bar=1 m+correction.

B \ A	21	12	39	153 R	354
	μ	μ	μ	μ	μ
21	0	−0.09 (−.08)	−45.50 (−45.41)	+2.40 (+2.37)	−41.61 (−41.69)
12	+0.09	0	−45.35 (−45.33)	+2.47 (+2.45)	−41.60 (−41.61)
39	+45.50	+45.35	0	+47.76 (+47.78)	+3.62 (+3.72)
153 R	−2.40	−2.47	−47.76	0	−44.04 (−44.06)
354	+41.61	+41.60	−3.62	+44.04	0
Σ	+84.80	+84.39	−142.23	+96.67	−123.63
Σ/5	+16.96	+16.88	−28.45	+19.33	−24.73
Bar−21	0	−.08	−45.41	+2.37	−41.69
Corr. to bar	+3.60	+3.52	−41.81	+5.97	−38.09

The confirmation of the difference in length found in 1928 between meters 12 and 21 is very reassuring and a change in length of meter 39 from a correction of −42.02 μ in 1928 to −41.81 μ in 1930 was not inconsistent with the change in length in this bar ever since its first standardization.

A comparison of the BIPM and NBS corrections for the platinum-iridium meter bars is given in table 5.

TABLE 5.—*Comparison of* BIPM *and* NBS *corrections for the platinum-iridium meter bars*

Length of bar = 1 m + correction.
Orig. = BIPM values at 0° C used as basis for certificates, and values at 20° C computed from BIPM certificates.
NBS 1933 = 1928 NBS observations reduced to 0° C and to 20° C using BIPM 1933 values for coefficients of expansion and for length of meter 27. (See table 1.)

Meter bars	Correction at 0° C				Correction at 20° C			
	BIPM		NBS	NBS−BIPM	BIPM		NBS	NBS−BIPM
	Orig.	1933	1933	1933	Orig.	1933	1933	1933
	μ	μ	μ	μ	μ	μ	μ	μ
27	−1.55	−1.47	−1.47	.00	+171.9	+171.67	+171.67	.00
21	+2.45	+3.42	+3.60	+.18	+176.2	+176.56	+176.74	+.18
12 (1874)	+3.32	+3.34	+3.48	+.14	+176.4	+176.09	+176.23	+.14
4 (1874)	−5.9	−5.13	−5.31	−.18	+167.6	+167.62	+167.44	−.18

Several points should be noted in considering these data. The first scientific paper of the NBS, Recomparison of the United States Prototype Meters, by Louis A. Fischer, discussed measurements made by him at the BIPM in 1903. These comparisons with laboratory working standards at approximately 17° C indicated that the length of the United States Prototype Meter 27 at 0° C was not in accord with the values determined in 1888 and given in the original BIPM certificate. An apparent shortening of 0.4 μ was shown. Mr. Fischer pointed out the desirability of recomparing meter 27 with the International Prototype Meter and with other national prototypes, and suggested the desirability of redetermining the coefficient of thermal expansion. It was decided at the NBS to continue to use the original certified value of meter 27 until further comparisons had been made. That has been done.

The measurements recently completed at the BIPM showed that a redetermination of the coefficients was indeed necessary, and that laboratory standards in use there had changed in length so that when meter 27 was compared in 1903 and meter 4 in 1907, erroneous values were reported. As comparisons were usually made at temperatures between 15 and 20° C and values were certified at 0° C, the situation is somewhat complicated. Changes in values for the corrections do not in such cases necessarily indicate changes in the physical lengths of the standards. But the change in the correction for meter 21, which has probably had more laboratory use than any other platinum-iridium standard distributed by the BIPM, is probably in large part an actual change in length. On the other hand it seems that meter 27 has not changed in length. Although the BIPM 1933 values for meters 12 and 4 are merely revisions of the results of comparisons of 1888 and 1907, respectively, the values found by the NBS indicate that the actual physical length of each of these standards may have changed, meter 12 lengthening by 0.1μ and meter 4 shortening by 0.2μ.

The adoption and use by the NBS of the new certificate for meter 27 is obviously the correct procedure. In the case of the other three platinum-iridium meters, however, it has been decided to use the results of the NBS intercomparisons. Actually the differences

between the BIPM and the NBS values are negligible as compared with the accuracy of length comparisons generally certified by a national laboratory.

2. CALIBRATION OF METER 39 TO DECIMETERS

Three calibrations of invar meter 39 have been made in recent years to determine the correction to the decimeter intervals. These measurements were made on the same longitudinal comparator as was used for the total lengths of the meter bars, using the method described in Bureau of Standards Circular 329, Calibration of a Divided Scale, by Lewis V. Judson. The results are given in table 6.

Table 6.—*Calibrations of meter 39 to decimeters*

Elements of calibration=corrections to intervals assuming that the total length of meter 39 is correct. Values given in columns 2, 3, and 4, are the measured values obtained in 1928, 1931, and 1932, respectively. Relative length of interval=nominal length+element of calibration.

1	2	3	4
Interval	Elements of calibration		
Decimeters:	μ	μ	μ
0 to 1	−7. 20	−7. 38	−7. 38
0 to 2	−1. 41	−1. 48	−1. 52
0 to 3	+1. 02	+. 95	+1. 06
0 to 4	+2. 29	+2. 25	+2. 30
0 to 5	−1. 59	−1. 56	−1. 54
0 to 6	+2. 27	+2. 25	+2. 25
0 to 7	+. 05	−. 02	+. 02
0 to 8	−5. 75	−5. 85	−5. 84
0 to 9	+. 97	+. 95	+. 86
0 to 10	. 00	. 00	. 00

III. CALIBRATION OF THE DECIMETER BARS

The calibration of the decimeter bars was carried out in two steps: first, the determination of the total lengths, and second, the calibration of the subintervals.

1. DETERMINATION OF TOTAL LENGTH

Two series of measurements were made in the determination of the total lengths of the decimeter bars. The first series consisted of an intercomparison of the seven decimeter bars among themselves, each bar being compared with each of the others. These measurements were all made on the same longitudinal comparator as was used in the intercomparison of the meters. The mean temperature was 23.65° C, and values were reduced to this temperature, using the new values given by the BIPM for the coefficients. No significant change would have been made, however, if the values originally certified had been used. These results are given in tables 7 and 8.

TABLE 7.—*Comparison of total lengths of decimeter bars at 23.65° C.*

Differences reduced to the mean temperature 23.65° C using values of coefficient of expansion given in column 10 of table 1.
A−B=Δ*L*.

Bars compared	Observed difference Δ*L*	Probable error	Observed temperature, *T*
A B	μ	μ	°C
20 minus 10	−6.26	±0.041	23.87
24 minus 10	+11.19	±.035	24.04
43 minus 10	−5.62	±.052	23.86
56 minus 10	−.99	±.021	23.90
57 minus 10	+8.04	±.042	23.98
61 minus 10	+20.79	±.033	23.26
24 minus 20	+17.28	±.044	24.00
43 minus 20	+.56	±.043	24.12
56 minus 20	+5.29	±.026	23.78
57 minus 20	+14.17	±.047	24.02
61 minus 20	+27.40	±.044	23.52
43 minus 24	−16.90	±.037	23.58
56 minus 24	−11.93	±.027	23.60
57 minus 24	−2.99	±.024	23.56
61 minus 24	+10.18	±.033	23.48
56 minus 43	+4.70	±.046	23.54
57 minus 43	+13.87	±.030	23.85
61 minus 43	+26.64	±.028	23.52
57 minus 56	+8.79	±.043	23.84
61 minus 56	+22.20	±.031	23.57
61 minus 57	+13.05	±.025	22.68

TABLE 8.—*Calculation of the most probable values of the differences in the total lengths of the decimeter bars at 23.65° C (first series)*

Observational values of (A−B) are taken from table 7.
Calculated ("most probable") values of (A−B), given in parentheses, are obtained by taking differences of Σ/7, e. g., decimeter 20−decimeter 10=−10.14−(−3.88)=−6.26μ.

B \ A	10	20	24	43	56	57	61
	μ	μ	μ	μ	μ	μ	μ
10	0	−6.26 (−6.26)	+11.19 (+11.04)	−5.62 (−5.72)	−0.99 (−.97)	+8.04 (+8.00)	+20.79 (+21.06)
20	+6.26	0	+17.28 (+17.30)	+.56 (+.54)	+5.29 (+5.29)	+14.17 (+14.26)	+27.40 (+27.32)
24	−11.19	−17.28	0	−16.90 (−16.76)	−11.93 (−12.01)	−2.99 (−3.04)	+10.18 (+10.02)
43	+5.62	−.56	+16.90	0	+4.70 (+4.75)	+13.87 (+13.72)	+26.64 (+26.78)
56	+.99	−5.29	+11.93	−4.70	0	+8.79 (+8.97)	+22.20 (+22.03)
57	−8.04	−14.17	+2.99	−13.87	−8.79	0	+13.05 (+13.06)
61	−20.79	−27.40	−10.18	−26.64	−22.20	−13.05	0
Σ	−27.15	−70.96	+50.11	−67.17	−33.92	+28.83	+120.26
Σ/7	−3.88	−10.14	+7.16	−9.60	−4.85	+4.12	+17.18

The second series consisted of direct comparisons of each of the four decimeter bars belonging to the NBS with two intervals, 20 to 30 centimeters and 70 to 80 centimeters on invar meter 39. From these direct comparisons with meter 39, results were obtained as shown in column 2 of table 9.

TABLE 9.—*Total length of decimeter bars*

Length of bar=1 dm+correction

Decimeter number	Corrections at 23.65° C	
	NBS direct comparisons	NBS adjusted values
1	2	3
	μ	μ
10	-------	+5.16
20	-------	−1.10
24	+16.32	+16.20
43	−.67	−.56
56	-------	+4.19
57	+12.89	+13.16
61	+26.49	+26.23

By combining the data in table 8 with the values in column 2 of table 9, adjusted corrections at 23.65° C were obtained for each of the bars. These values are given in table 9, column 3.

Table 10 shows the NBS observations reduced to 0° C and to 20° C, using the old certified values for the coefficients, and also the new values. The original certified corrections are also shown for comparison.

TABLE 10.—*Comparison of* BIPM *and* NBS *corrections for the decimeter bars*

Length of bar=1 dm+correction. For values of coefficients see table 1

Decimeter number	Corrections at 0° C			Corrections at 20° C		
	BIPM (certificate)	NBS		BIPM (certificate)	NBS	
		Using old coefficients	Using new coefficients		Using old coefficients	Using new coefficients
	μ	μ	μ	μ	μ	μ
10	+2.0	+2.89	+2.73	+3.9	+4.81	+4.80
20	−4.5	−3.37	−3.53	−2.6	−1.45	−1.46
24	−1.8	−3.24	−3.23	+14.7	+13.22	+13.22
43	−4.0	−2.83	−2.99	−2.1	−.91	−.92
56	+.4	+1.92	+1.76	+2.3	+3.84	+3.83
57	−6.0	−6.28	−6.27	+10.5	+10.18	+10.18
61	−3.2	−3.79	−3.78	+22.1	+21.54	+21.53

2. CALIBRATION OF THE SUBINTERVALS

The calibration of subintervals was made for the same intervals as was done for the original BIPM certificates, namely, for the centimeter intervals, for the millimeters of the first centimeter and for the tenth-millimeters of the specially graduated millimeter.

Most of the measurements were made on the longitudinal comparator, although some of the work at the beginning was done on a linear dividing engine used as a comparator.

The general scheme used in the original calibration at the International Bureau was carried out. Guillaume's method[3] for the reduction of the observations was employed as in the work done at Sèvres.

A good idea of the results of the calibration will be obtained by considering the calibrations of decimeter 43, expressing the results as "element of calibration", that is, the corrections which the subintervals have if the length of the whole interval is considered as correct, as will be clear from a consideration of table 11.

TABLE 11.—*Elements of calibration of decimeter 43*

Elements of Calibration=corrections to intervals assuming that the total length (100, 10, or 1.0 mm) is correct.
Relative length of interval=nominal length+element of calibration.

Interval	BIPM	NBS values using auxiliary decimeter indicated						
		Dm 10	Dm 20	Dm 24	Dm 56	Dm 57	Dm 61	Mean
Millimeters:	μ	μ	μ	μ	μ	μ	μ	μ
0 to 10	+1.0	+0.92	+0.91	+0.96	+1.04	+0.82	+1.11	+0.96
0 to 20	+1.1	+.95	+.95	+.84	+1.00	+.72	+.98	+.91
0 to 30	+.6	+.55	+.46	+.42	+.58	+.30	+.61	+.49
0 to 40	+.3	+.05	+.24	.00	+.21	−.12	+.05	+.07
0 to 50	+.6	+.39	+.67	+.35	+.53	+.13	+.41	+.41
0 to 60	+.7	+.37	+.62	+.33	+.55	+.26	+.43	+.43
0 to 70	−.1	−.39	.00	−.34	−.13	−.48	−.41	−.29
0 to 80	−1.4	−1.67	−1.33	−1.75	−1.41	−1.81	−1.66	−1.60
0 to 90	−1.0	−1.22	−.83	−1.18	−.89	−1.07	−1.06	−1.04
0 to 100	.0	.00	.00	.00	.00	.00	.00	.00
0 to 1	−.2	−.06	−.02	−.10	−.15	−.05	−.06	−.07
0 to 2	+.1	+.10	+.22	+.15	+.14	+.14	+.16	+.15
0 to 3	−.1	.00	+.02	+.04	+.07	+.05	+.09	+.04
0 to 4	+.1	+.01	+.15	+.08	+.12	+.10	+.20	+.11
0 to 5	−.1	−.11	−.03	−.07	−.07	−.13	+.01	−.07
0 to 6	−.1	+.04	+.01	−.07	−.04	−.10	+.04	−.02
0 to 7	+.5	+.30	+.39	+.32	+.41	+.38	+.49	+.38
0 to 8	+.4	+.42	+.43	+.42	+.42	+.30	+.41	+.40
0 to 9	+.3	+.24	+.26	+.28	+.24	+.19	+.32	+.26
0 to 10	.0	.00	.00	.00	.00	.00	.00	.00
0 to 0.1	.0	−.10	−.10	+.06	−.07	+.21	+.10	+.02
0 to 0.2	−.2	−.18	−.21	−.16	−.15	−.17	+.10	−.13
0 to 0.3	−.4	−.27	−.89	−.09	−.71	−.01	+.02	−.32
0 to 0.4	−.9	−.62	−.82	−.55	−.81	−.72	−.66	−.70
0 to 0.5	−.8	−.66	−.79	−.70	−.80	−.75	−.57	−.71
0 to 0.6	−1.2	−1.04	−1.14	−1.11	−1.20	−1.22	−1.07	−1.13
0 to 0.7	−1.2	−.96	−1.16	−1.06	−1.09	−1.24	−1.04	−1.09
0 to 0.8	−.9	−.64	−.90	−.80	−.84	−.75	−.79	−.79
0 to 0.9	−.4	−.17	−.18	−.36	−.25	−.38	−.20	−.26
0 to 1.0	.0	.00	.00	.00	.00	.00	.00	.00

The interval is given in the first column, the element of calibration as obtained in the BIPM calibrations in the second column, the several elements of all calibrations as obtained at the NBS by comparison with each of six decimeters, and the mean of these six values in the following columns.

Although there are variations between the several sets of values, these are not excessive when the conditions of the lines and surfaces of these decimeter bars are taken into consideration. They are discussed further in the next section. The mean value of the element of calibration is certainly sufficiently accurate for our purposes.

The lengths of the subdivided millimeters were determined from known millimeter intervals on these decimeter bars. The results, compared with the original values, appear in table 12.

[3] C. E. Guillaume, *L'étalonnage des échelles divisées*. Travaux et Mémoires du Bureau International des Poids et Mesures, **13**, 1–54 (1907).

TABLE 12.—*Corrections to subdivided millimeters of decimeter bars*

Length of interval=1 mm+correction

Decimeter number	Correction at 0°C	
	BIPM certificate	NBS determinations
	μ	μ
10	-----------	+0.28
20	-----------	+.14
24	−0.2	+.20
43	−1.0	−.59
56	-----------	−.50
57	−.1	−.21
61	−1.6	−.91

IV. SOURCES OF ERROR

The more important sources of error in all comparisons of line standards of length are these: (1) an uncertainty regarding the actual temperature of the bars as to whether or not each bar is in thermal equilibrium at the temperature indicated by the thermometer; (2) difficulties in setting upon the lines, owing to defects in the lines and in the polished surfaces on which they are ruled; and (3) personal errors in setting on the lines, resulting in differences between the results obtained by different observers. These differences are enhanced by the defects in the lines and surfaces.

Sears and Barrell of the National Physical Laboratory, Teddington, England, have recently stated:[4]

> In the authors' opinion the average accuracy of repetition generally attainable in the comparison of two line standards of the best class, such, for example, as the Prototype Metre and its National Copies, is of the order of 0.25×10^{-6} M, which corresponds to a possible range of values of about ±1 part in 8,000,000.[5] Differences of this order of magnitude are liable to occur between the results of comparison of the same bars by different observers.

From the context it is obvious that they regard the condition of the lines and surfaces as an important factor in causing such variations. All of which agrees with the experience of the NBS.

In the comparison of the meter bars the smallness of the residuals, that is, the differences between the observed and computed values of table 3, indicates that the bars were in good thermal equilibrium at the time the observations were made. The comparator is in a basement room; several hours elapsed between sets of measurements; and precautions were taken to maintain uniform temperature.

In the calibration of the decimeter bars the effect of temperature is very small. It will be noted that in the case of the invar bars, an error of 0.1° C in the temperature measurement would result in an error in the length of one of these bars of only 0.01μ. Even in the case of the nickel bar, 0.1° C corresponds to 0.1μ. Special care was used to insure that the bars and thermometers had come to a condition of equilibrium. It is not likely, therefore, that there are in this work any errors of significance ascribable to variations in temperature, or to errors in temperature measurement. A comparison of tables 3 and 4, representing measurements by different observers,

[4] J. E. Sears, Jr., and H. Barrell, *Determinations of the fundamental standards of length in terms of the wavelength of light,* Phil. Trans. Roy. Soc. London, **233**, 173 (1934).
[5] This apparently should be ± 1 part in 2,000,000.

with three bars in common, indicates that these measurements of the meter bars are free from any large personal errors.

The probable error of each of the values of the elements of calibrations of meter 39 in table 6 is approximately 0.03μ.

That there are relatively large errors in our work with the decimeter bars is readily explained by the imperfections of the lines and surfaces. The probable error of each of the calculated differences in total length of the seven bars, as determined by their direct intercomparisons (table 8), is $\pm$ 0.05μ. By excluding the nickel bar 61 because the residuals on this bar are seen to be large, a probable error of $\pm$ 0.04μ is obtained for the other six. Using the four invar bars, "best values" for the differences in length are obtained differing a few hundredths of a micron from those listed above but the probable error is only 0.02μ. However, consideration of the character of the lines and of the surfaces on which they were ruled has led us to the conclusion that measurements of the total length of these bars to better than about 0.05μ are without real meaning. The entire series of measurements with all seven bars has therefore been retained, and the relative results of the intercomparisons at 23.65° C are considered correct to about 0.1μ.

The chief source of error in the final value for the total lengths of the decimeter bars (columns 3, 4, 6, and 7 of table 10) is in the determination of the absolute values of the four decimeter bars, 24, 43, 57, and 61, belonging to the NBS, but careful study of the results leads to the conclusion that the final correction to the total lengths of each of the seven bars is not in error by more than 0.2μ, and that in most cases it is not in error by more than 0.1μ.

To what extent the differences between the old and the new values represent changes in the actual length of the bars, and to what extent they represent a greater accuracy in the newer measurements, it is hardly possible to state. Undoubtedly some of the changes are real changes in the length of the bars. As the measurements at the NBS were made with precise comparators of modern design, with high-power microscopes, and under such conditions as to yield accurate results, it is believed that the NBS is justified in adopting these new determinations as representing the true values at the time of measurement more nearly than could any value given by a certificate based upon measurements made one or more decades earlier.

In the calibration of the subintervals of the decimeter bars (see table 11 for decimeter 43), there are variations amounting to as much as 0.5μ. That these are caused by imperfections in the lines and surfaces of the decimeter bars, and not by the comparator, is demonstrated, not only by the measurements on the meter bars, but more especially by several sets of comparisons on decimeter scales ruled by C. G. Peters of the interferometry section on highly polished and almost flawless surfaces of bars of stainless steel. Those lines were very sharp and straight and had even edges, and with those scales measurements consistent to about 0.05μ were made. Therefore, with improvements in the surfaces and lines of line standards and commercial scales, this comparator is capable of a corresponding reduction in the uncertainty in the measurements.

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V. CONCLUSION

From the extensive calibrations reported in this paper, the National Bureau of Standards is enabled to adopt precise values for the corrections for its meter and decimeter bars. These values are all based on the values assigned to the United States Prototype Meter 27 by the Bureau International des Poids et Mesures, and on the latest and most precise values for the coefficients of thermal expansion of the several bars.

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